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Your Roll No.

B.Sc. (H) Computer Science / IV Sem. B
Paper 404 – Differential Equations
(Admissions of 2001 and onwards)

Time : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately
on receipt of this question paper.)

Question number 1 is compulsory and attempt any 4
questions from question number 2 to question no. 7.

Symbols have their usual meaning.

1. (i) Show that $y = \frac{\sin x}{x}$ is a solution of the
differential equation

$$xy'' + 2y' + xy = 0$$

Find a linearly independent solution by reducing
the order.

- (ii) Use the method of variation of parameters to find a
general solution of the differential equation

$$4x^2 y'' + 8xy' - 3y = 7x^2 - 15x^3$$

[P.T.O.]

(iii) Show that $\int x^{\nu} J_{\nu-1}(x) dx = x^{\nu} J_{\nu}(x) + C$
Where $J_{\nu}(x)$ is the Bessel function.

(iv) Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre's polynomials.

(v) Prove that if $f(t)$ has the Laplace transform $F(S)$ (where $S > K$), then $e^{at} f(t)$ has the transform $F(S-a)$ (where $S-a > K$), and hence find the transform of $\sinh t \cdot \cos t$.

(vi) Graph the function $t^2 U(t-1)$ and find its Laplace transform.

(vii) Find the inverse Laplace transform of

(a) $\frac{S}{S^2 + S + \frac{1}{2}}$

(b) $\frac{1}{S^2} \left(\frac{S-1}{S+1} \right)$ 7×5

2. (a) Solve the initial value problem
 $y'' - y' - 2y = 0, y(0) = -4, y'(0) = -17$

(b) Find a particular integral and hence find a general solution of the differential equation
 $y'' + 2y' + y = e^{-x} \cos x$ 5+5

3. (a) Solve the differential equation
 $xy'' - y' = (3+x)x^2 e^x$ 5+5

(b) Find the power series solution of the differential equation
 $xy' - 3y = K$
where K is a constant.

4. (a) Prove that $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} [(x^2 - 1)^n]$ 5+5

(b) Prove the following

(i) $J_{\frac{3}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left(\frac{\sin x}{x} - \cos x \right)$

(ii) $J_{-\frac{3}{2}}(x) = -\sqrt{\frac{2}{\pi x}} \left(\frac{\cos x}{x} + \sin x \right)$

5. (a) Solve the differential equation
 $(x^2 - x)y'' - xy' + y = 0$
using Frobenius method. 5+5

(b) Determine the radius of convergence of the power series :

(i) $\sum_{m=0}^{\infty} \frac{1}{3^m} (x-3)^{2m}$

$$(ii) \sum_{m=0}^{\infty} m^m x^m$$

6. (a) Solve the system of differential equation using
Laplace transform 5+5

$$y_1' = -y_1 + y_2$$

$$y_2' = -y_1 - y_2$$

$$y_1(0) = 1, y_2(0) = 0$$

- (b) Find the transform of the function

$$f(t) = \begin{cases} 2 & \text{if } 0 < t < \pi \\ 0 & \text{if } \pi < t < 2\pi \\ \sin t & \text{if } t > 2\pi \end{cases}$$

7. (a) Solve the integral equation

$$y(t) = \sin 2t + \int_0^t y(\tau) \sin 2(t-\tau) d\tau$$

- (b) Solve the following differential equation using
Laplace transform

$$y'' + y = 2t$$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{2}\pi$$

$$y'\left(\frac{\pi}{4}\right) = 2 - \sqrt{2} \quad \text{5+5}$$